

Logarithmic entanglement lightcone in many-body localized systems

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We theoretically study the response of a many-body localized system to a local quench from a quantum information perspective. We find that the local quench triggers entanglement growth throughout the whole system, giving rise to a logarithmic lightcone. This saturates the modified Lieb-Robinson bound for quantum information propagation in many-body localized systems previously conjectured based on the existence of local integrals of motion. In addition, near the localization-delocalization transition, we find that the final states after the local quench exhibit volume-law entanglement. We also show that the local quench induces a deterministic orthogonality catastrophe for highly-excited eigenstates, where the typical wave-function overlap between the pre- and post-quench eigenstates decays *exponentially* with the system size.

I. INTRODUCTION

It is well known that quantum interference induced by the elastic scattering of random impurities can localize a quantum system. This leads to the celebrated Anderson insulator with an absence of diffusion and zero DC conductivity [1]. Without interactions, generic elastic disorder localizes all single-particle quantum states in one and two dimensions [2]. The fate of isolated interacting quantum systems with strong disorder has attracted considerable recent attention, both theoretical [3–12] and experimental [13–16], under the rubric of many-body localization (MBL). MBL phases have a number of remarkable properties, ranging from the violation of the eigenstate thermalization hypothesis [17–19] to emergent integrability [7, 20–23] and localization-protected orders that are normally forbidden by the Peierls-Mermin-Wagner theorem [22, 24–28]. Most interestingly, MBL systems are not self-thermalizing, which calls into question the generic applicability of quantum statistical mechanics to isolated systems without external baths. In addition, MBL systems are argued, via a variant of the Lieb-Robinson bound [29], to exhibit a logarithmic lightcone (i.e., information can propagate at most logarithmically in time) [30, 31], which is consistent with the logarithmic growth of entanglement following a global quench [7, 32–35]. This is in stark contrast to both thermal many-body systems, where entanglement grows ballistically [36, 37], and generic quantum systems without disorder, where an emergent linear lightcone can be proven [38–40] and even experimentally observed [41–44].

So far, the response of MBL systems to a local quench remains rarely explored in contrast to the global quench which has been studied extensively. For metallic systems, an arbitrarily weak local quench can substantially modify the structure of the many-body ground state. This leads to the Anderson orthogonality catastrophe (OC) where the post-quench ground state becomes asymptotically orthogonal to the original one [45, 46]. Very recently, a new idea of statistical OC was introduced [47] and an exponential-, rather than algebraic-, law has been

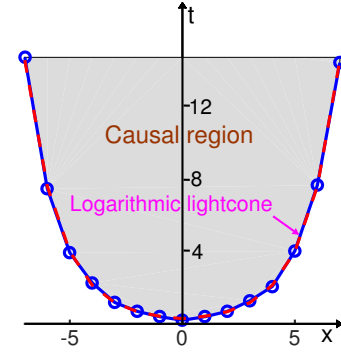


FIG. 1: Logarithmic lightcone extracted from the local quench triggered entanglement growth. We begin with highly-excited states (obtained by DMRG-X) and calculate $S_x(t)$ from TEBD (see text). The circled blue curve is determined by investigating when $S_x(t)$ starts to increase and the red dashed line is a logarithmic fit $x \sim \log t$ to the data.

established for the ground states of both single-particle (without interaction) and many-body (with interaction) localized systems [48]. Yet, OC for highly-excited states of a localized system remains largely unexplored. In addition, the wavefunction dephasing dynamics following a local quench is still poorly understood for highly-excited states, except for the general expectation that a local quench cannot spread out energy or particle density. In this paper we provide insight into the physics of MBL by investigating OC for highly-excited states and probing the entanglement dynamics subsequent to a local quench. We address the following questions. Can a local quench, which only changes an intensive amount of energy, fundamentally change the wavefunction of a MBL system? How does quantum information propagate in a MBL phase? Can we directly probe the logarithmic lightcone?

We consider an initial disordered interacting Hamiltonian H_I (all eigenstates of H_I are localized), and add a local quench V_0 to H_I at time $t = 0$. We find that the

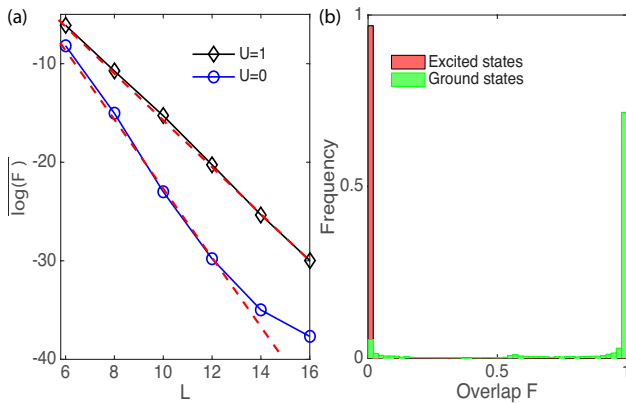


FIG. 2: Wave-function overlaps between the pre- and postquench eigenstates in localized systems. (a) Exponential OC of the highly-excited eigenstates, at $W = 15$ and $v_0 = 6$. Here $|\psi_I\rangle$ and $|\psi_F\rangle$ are chosen to be the excited states in the middle of the spectrum of H_I and H_F , respectively. The red dashed lines are linear fits to the data. The deviation of the blue curve ($U = 0$) from the linear fit at large L is due to the machine precision limit. (b) Histogram of the overlaps of the ground and excited states, respectively, at $W = 15$, $v_0 = 6$, $U = 1$ and $L = 10$. Unlike ground state OC which happens stochastically with a probability $\sim \frac{v_0}{2W}$ [48], OC for highly-excited state occurs deterministically (with probability one in the thermal dynamic limit).

typical overlap between the pre- and post-quench excited states is suppressed *exponentially* in the system size, inducing a deterministic OC in the thermodynamic limit. Combining extensive exact diagonalization and density matrix renormalization group (DMRG) calculations, we show that the local quench causes dephasing and triggers an entanglement growth throughout the system. By comparing the entanglement entropy dynamics for different bipartitions, we show that the local quench propagates quantum information through the chain logarithmically in time. This explicitly demonstrates, from first principles, the existence of the logarithmic lightcone (Fig. 1) conjectured in MBL systems, without any assumptions about the local integrals of motion of the MBL system [30]. Moreover, near the localization-delocalization transition, we find that for highly excited states the local quench can trigger an *unbounded* entanglement growth with an asymptotic volume-law scaling in the long time limit.

II. MODEL HAMILTONIAN

We consider the generalized interacting spinless one-dimensional (1D) Anderson model [1]

$$H_I = \sum_{j=-L/2+1}^{L/2} -J(c_j^\dagger c_{j+1} + h.c.) + \mu_j n_j + U n_j n_{j+1}, \quad (1)$$

where c_j (c_j^\dagger) are fermionic annihilation (creation) operators, $n_j = c_j^\dagger c_j$ is the corresponding fermion number operator, J is the nearest-neighbour hopping strength, $U > 0$ is the nearest-neighbor interaction, and μ_j is the onsite disorder potential drawn independently from a uniform random distribution over $[-W, W]$. We set $J = 1$ as the unit of energy and consider half filling. The number of disorder realizations ranges from 10^5 ($L = 6$) to 10^3 ($L = 50$). The Hamiltonian in Eq. (1) has been explored in the context of single-particle [2] and many-body localization [6, 12, 49]. In the noninteracting limit $U = 0$, the system is localized for any finite value of W [50]. For $U > 0$, it is now generally accepted that the system has a MBL transition at a finite W . With an interaction strength $U = 1$, the reported numerics suggests a critical disorder strength $W_c \gtrsim 7$ [6, 51].

III. ORTHOGONALITY CATASTROPHE FOR HIGHLY-EXCITED STATES

To study the effect of local perturbations on the eigenstates, we quench the model at $t = 0$ by adding a local perturbation $V_0 = v_0 c_0^\dagger c_0$ at site 0. The final Hamiltonian for $t > 0$ is $H_F = H_I + V_0$. We perform full diagonalization on both H_I and H_F and pair up their eigenstates $|\psi_I\rangle$ and $|\psi_F\rangle$ per the eigenvalue ordering. For each pair, we compute the many-body wavefunction overlap (fidelity) $F \equiv |\langle \psi_I | \psi_F \rangle|$ and study its asymptotic behavior as a function of the system size. As shown in Fig. 2(a), we find that the typical wave-function overlap for highly-excited states $F_{typ} \equiv \exp(\overline{\log F})$ decays exponentially with the system size

$$F_{typ} \sim \exp(-\beta L),$$

similar to the case of ground states [48]. Here, $\beta > 0$ is related to the localization length ξ and the strength v_0 of the local quench. This exponential decay is universal for localized systems, independent of whether it is single-particle or many-body localized. It originates from the nonlocal charge transfer process induced by a local quench in localized systems [48]. The exponential decay can also be understood intuitively from the local-integrals-of-motion description of MBL. Within this description, the many-body eigenstates are product states of the local conserved quantities, which commute with the Hamiltonian and have exponentially small tails on operators far away. The pre- and post-quench eigenstates have many different eigenvalues of the local conserved quantities, giving rise to an exponential suppression of the typical overlap between them.

However, it is worth noting that the OC of the highly-excited states differs from that of the ground states in a statistical sense. For ground states, the OC occurs with a probability proportional to the strength of the perturbation [48]. While for highly-excited states, there will

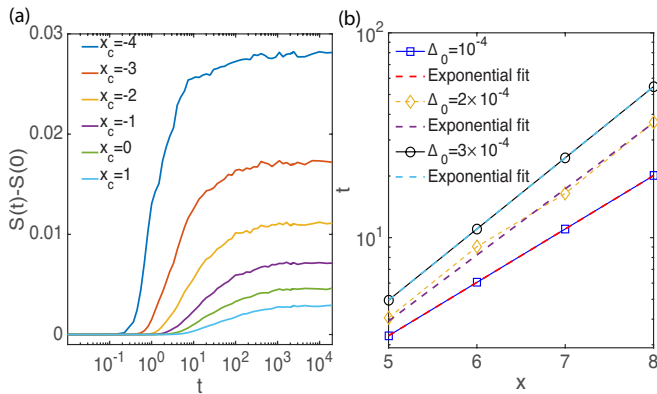


FIG. 3: Exact diagonalization calculations of entanglement dynamics introduced by a local quench and the extracted lightcones for information propagation. (a) Deep localized region with $W = 15$, $U = 1$, $L = 12$ and $v_0 = 6$. We begin with a highly-excited state at the middle of the spectrum. x_c indicates the location where we cut the system into two subsystems A and B (cutting the bond between site x_c and $x_c + 1$). The local quench is added at the left end (site -5 in our notation) of the chain. (b) The extracted logarithmic lightcones for different threshold values. For a given threshold value Δ_0 , the corresponding logarithmic lightcone is obtained by studying when $S_x(t)$ starts to increase to pass Δ_0 (see the text).

always be a multi-particle rearrangement and we have a deterministic catastrophe with probability one in the thermodynamic limit $L \rightarrow \infty$ [47]. This distinction is clearly visible in Fig. 2(b).

IV. ENTANGLEMENT GROWTH

The OC implies that the new eigenstates after the quench are typically orthogonal to the corresponding initial ones, but it does not necessarily imply an essential change in their internal properties such as the entanglement entropy. In a metallic system without disorder, it has been proven recently that the logarithmic scaling with the system size of the ground state entanglement entropy remains intact in spite of the Anderson OC induced by a local perturbation [52]. In the localized system considered here, notwithstanding the exponential OC, the postquench Hamiltonian is still localized and thus its eigenstates still have area-law entanglement [26]. In neither cases does the local quench change the static entanglement scaling properties of the Hamiltonian eigenstates. However, as we show below, the local quench exerts a strong influence on the entanglement dynamics of the highly-excited states in the MBL system.

To study the entanglement dynamics after the quench, we take an eigenstate $|\psi_I\rangle$ of H_I and evolve it under the postquench Hamiltonian H_F : $|\phi_F(t)\rangle = e^{-iH_F t}|\psi_I\rangle$. We note that this quench is distinct from a quench of the

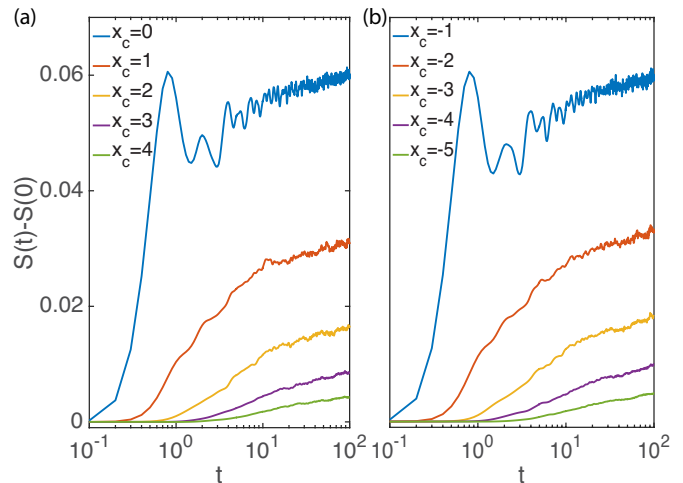


FIG. 4: The DMRG-X and TEBD calculations of entanglement growth after a local quench starting from a highly-excited eigenstate $|\psi_I\rangle$ of H_I , for large system sizes $L = 50$, with $U = 1$, $W = 15$, and $v_0 = 6$. Here, the local quench is added in the middle of the chain (site 0). (a) and (b) show the entanglement growth $S(t) - S(0)$ for the right and left half chains, respectively. The logarithmic lightcone plotted in Fig. 1 is obtained from the same data shown here.

wavefunction [28]. We study the time dependence of the von Neumann entropy of $|\phi_F(t)\rangle$ between two subregions A and B:

$$S(t) = -\text{Tr}[\rho_A(t) \log \rho_A(t)],$$

where $\rho_A(t) = \text{Tr}_B[|\phi_F(t)\rangle\langle\phi_F(t)|]$ is the reduced density matrix of the subsystem A. To calculate $S(t)$, we use exact diagonalization for small system sizes $L \leq 16$. In Fig. 3 (a), we plot $S(t)$ for different bipartitions for the region deep in the MBL phase ($W = 15$). We find that the entanglement indeed increases, even for bipartitions far away from the local quench. This can be explained as follows. Deep in the MBL phase ($W = 15$ and $U = 1$), although the wavefunctions are localized (product states in the so-called ‘l-bit’ formalism [7, 20]), the local quench still causes dephasing because of the exponentially weak couplings between distant l-bits [34] and leads to entanglement growth throughout the whole system as shown in Fig. 3 (a). This is in contrast to the non-interacting Anderson model, where a zero-velocity Lieb-Robinson bound [53] simply forbids the spread of entanglement and restricts the effect of the local quench to within a few localization lengths even in the infinite time limit [54]. Interaction enables a spreading of entanglement in the localized system.

For larger systems (up to $L = 50$), we use the recently developed DMRG-X [55] to obtain the excited states. The DMRG-X algorithm designed for 1D MBL systems can be seen as a variant of the conventional ground-state DMRG algorithm based on the matrix product state (MPS) representation [56, 57]. Their procedure is sim-

ilar, but the key difference is that when we update local matrices, we update each local matrix with a new one that maximizes the overlap with the previous MPS, rather than the one corresponding to the minimal energy. We then use the time evolving block decimation (TEBD) method [58, 59] to perform the time evolution and compute $|\phi_F(t)\rangle$ and $S(t)$. In our calculations, we use the fifth-order Suzuki-Trotter decomposition of the short time propagator and the maximal bond dimension (where we truncate the MPS) is chosen as $\chi_{\max} = 30$. To control the error, we examine the typical variances $\sigma^2 = \langle H^2 \rangle - \langle H \rangle^2$ (less than 10^{-8}) in the DMRG-X and check the neglected weight for each truncation ($\sim 10^{-6}$) in the TEBD calculations.

In Fig. 4, we plot the entanglement dynamics, calculated by the DMRG-X and TEBD techniques, for different bipartitions with a larger system size $L = 50$. Similar to the exact diagonalization results plotted in Fig. 3(a), we again find entanglement growth. Another observation is that we can use $S(t)$ as a function of the partition distance from the local quench (Fig. 4(a)) to provide a rough estimate of the many-body localization length ξ . Note that the initial rapid “boost” (until $Jt \sim 1$) and “jitter” behavior of $S(t)$ arise from the expansion of wave packets within a distance ξ of the local quench. However, the wave packet cannot diffuse beyond ξ . The influence of the quench may propagate beyond ξ only via dephasing, and the entanglement growth after $Jt \gtrsim \xi$ is purely due to interactions. Hence there is no rapid boost or jitter behavior of $S(t)$ for partitions that exceed a distance ξ from the local quench as shown by the $x_c = 2$ curve in Fig. 4(a). We therefore conclude from Fig. 4(a) that ξ is on the order of a couple of lattice spacings for $U = 1$.

V. SCALING OF THE $t \rightarrow \infty$ ENTANGLEMENT

In the previous section, we have shown that a local quench can trigger entanglement growth throughout the whole system. We now discuss the post-quench entanglement dynamics in the long time limit.

In Fig. 5 (a), we have plotted the saturated (i.e., $t \rightarrow \infty$) entanglement as a function of the distance d between the local quench and the entanglement cut. We find that the saturation values decrease exponentially as the distance increases. Moreover, as the disorder strength increases, the decrease of the saturation value steepens. This is consistent with the local integrals of motion description of MBL: the l -bits have exponential tails on locations far away. We thus expect $S_d(\infty) - S_d(0) \sim e^{-\alpha d/\xi}$, where $S_d(t)$ denotes the entanglement entropy for a bipartition where the cut location is d sites away from the local quench, and α depends on the details of the Hamiltonian. When increasing the disorder strength W , ξ decreases and thus the net entanglement growth at long time $S_d(\infty) - S_d(0)$ decreases

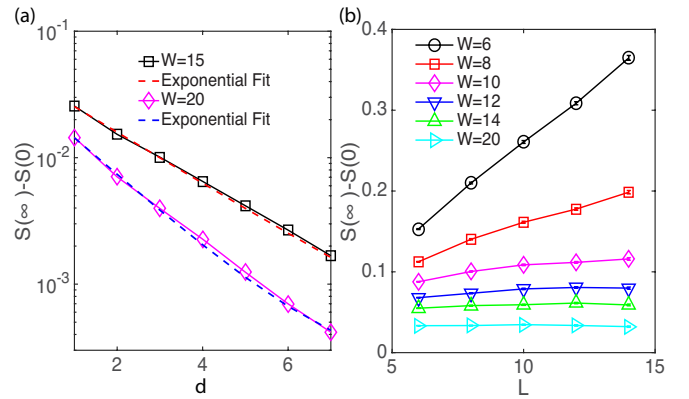


FIG. 5: (a) Final entanglement growth versus the distance between the local quench site and the entanglement cut. The results for two different disorder strengths ($W = 15, 20$) deep in the MBL phases are plotted. In the long-time limit, the net entanglement growths introduced by the local quench decay exponentially with the distance from the quench. Here, the other parameters are $U = 1$ and $v_0 = 6$. (b) Scaling of the saturated entanglement with the system size L for different W . Here, we consider a bipartition into two half chains of equal size. Deep in the localized region ($W = 14, 20$), the final states after the quench exhibit area-law entanglement. In contrast, near the localization-delocalization transition point ($W = 6, 8$), the local quench gives rise to an unbounded entanglement growth in the thermodynamic limit $L \rightarrow \infty$. For finite L , the saturation value of the entanglement scales linearly with the system size $S(\infty) \sim L$.

faster.

In Fig. 5 (b), the scaling of the saturated entanglement with the system size L is plotted. We find that deep in the localized region, while the local quench indeed induces a finite entanglement growth, the final post-quench states seem to still obey an area-law entanglement. The entanglement growth is finite even in the thermodynamic limit $L \rightarrow \infty$. This is in sharp contrast to the extensively-studied case of global quench [7, 32–35]. There, one starts with random product states and an unbounded logarithmic entanglement growth has been shown. In this case, the saturated entanglement follows a volume law even in the parameter regions very deep in the localized side. Interestingly, for the local quench studied in this paper, we find that near the MBL transition ($W_c \gtrsim 7$ [6, 51]), $S(\infty) - S(0)$ does not saturate in L and the local quench can also induce an unbounded entanglement growth when $L \rightarrow \infty$ in the thermal phase ($W = 6$). The saturated entanglement satisfies a volume law, as shown in Fig. 5 (b) for $W = 6$.

VI. LOGARITHMIC LIGHTCONE

We now turn to the important question of how fast the influence of the local quench can propagate. A modified Lieb-Robinson bound $\propto te^{-x}$ has been proposed for MBL

systems under certain assumptions [30]. This bound implies that information can propagate *at most* logarithmically, i.e. within a logarithmic lightcone. However, it is not clear whether this bound can be saturated or not in real MBL systems. Our local quench procedure provides an ideal setup to study this important bound and directly measure how fast the influence of the quench can spread without making any ad hoc assumption. To this end, we first calculate the entanglement dynamics for different bipartitions at a distance x from the quench site (denoted by $S_x(t)$), and then extract t as a function of x by studying when $S_x(t)$ begins to increase. In numerics, we impose a threshold Δ_0 on $\Delta_x(t) \equiv S_x(t) - S_x(0)$ and the function $t(x)$ is determined by the time when $\Delta_x(t)$ increases above Δ_0 : $t(x) = \min\{t | \Delta_x(t) \geq \Delta_0\}$. In the delocalized region ($W < W_c$), the influence of the local quench spreads out ballistically, we thus expect a linear lightcone. In contrast, in the localized region ($W > W_c$), we find a logarithmic lightcone $x \sim \log t$, as shown in Fig. 1 and Fig. 3(b). This *saturates* the modified Lieb-Robinson bound in the MBL phase. We stress that our observation of the logarithmic lightcone comes directly from the numerical data on sufficiently large system sizes, which does not rely on any assumption and is free from any significant finite size effects. It establishes that, unlike for the single-particle localized systems, the Lieb-Robinson bound for MBL systems cannot be further tightened to a bound constant in time, consistent with the analytical analysis in Ref. [60]. Our result is, however, free from any constraining assumptions and is established by numerically studying MBL systems of rather large sizes.

VII. CONCLUSION

We have studied the response of a MBL system to a local quench. We establish that a local quench can induce an *exponential* OC for highly-excited states in MBL systems, which is deterministic and has important implications on the post-quench entanglement dynamics. Using exact diagonalization and DMRG-X techniques, we demonstrate that in the interacting MBL system local quench triggers an entanglement growth throughout the whole system, in sharp contrast to the noninteracting localized situation where the entanglement is necessarily localized within a localization length. By numerically investigating $S_x(t)$, we explicitly establish the existence of a logarithmic lightcone conjectured in MBL systems. Our results indicate that quantum information propagates logarithmically in MBL systems and the modified Lieb-Robinson bound cannot be further tightened to a time-independent bound, in contrast to the case of single-particle localization. In addition, we have shown that near the localization-delocalization transition the local quench can trigger an *unbounded* entanglement growth

with an asymptotic volume-law scaling in the long-time limit.

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